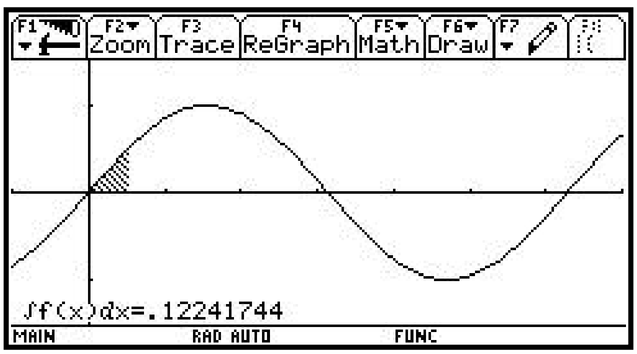
Lab 8: Area Functions and the FTC

Laboratory Experience

As we have seen, we can determine the area under a curve using a definite integral such as $\int_0^5 x^3 dx$. This gives the area under the graph of $y = x^3$ from $x = 0$ to $x = 5$. What if we also wanted to find the area under the curve from $x = 0$ to $x = 6$ or $x = 7$ or $x = 8$ and so on. For each of these endpoints we will have a new numerical value for the area. In this activity we will develop the concept of an "area function" where the output of the function depends on the upper limit of the definite integral.

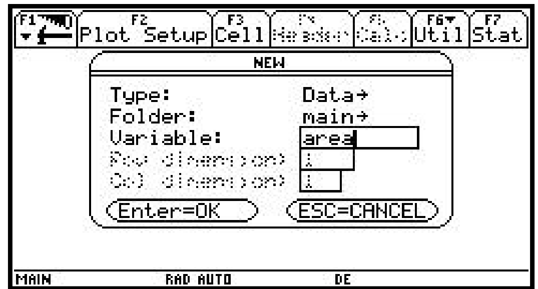
1. Consider the function $f(x) = \sin x$. We are going to look at the relationship between the upper limit of the definite integral and the area under the curve starting at $x = 0$. Record your observations/answers to these questions in a separate lab notebook.
   a. Use your area approximation command, **∫f (x) d x** under the F5 menu on the TI-89/92, to approximate the area under $f(x) = \sin x$ from $x = 0$ to 6.5 by increments of 0.5. Record your data. The first data point is shown in the table.

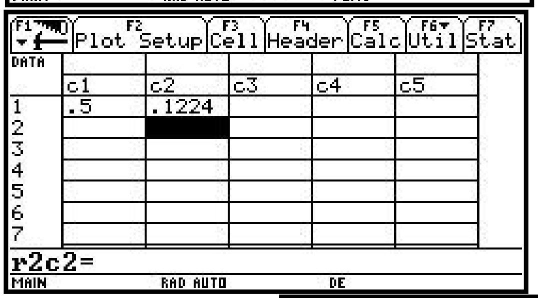


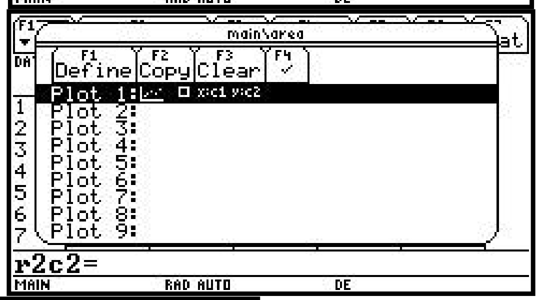
x	Area up to x
0.5	.12241744
1	
1.5	
2	
2.5	
3	
3.5	
4	
4.5	
5	
5.5	
6	
6.5	

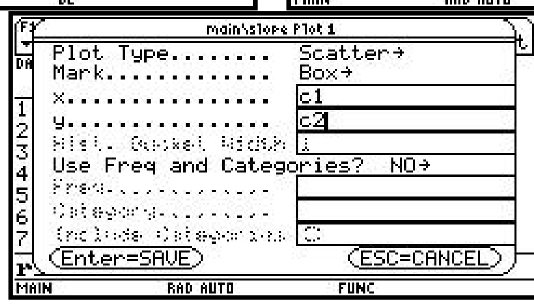
   b. After you have collected your data, enter the data into the **Data/Matrix Editor** on your graphing calculator or computer and perform a scatter plot. Follow the series of screen images below to accomplish the scatter plot on a TI-92plus/89 or

Voyage 200. Begin by pressing the APPS key and going to the menu shown below. Place your x data in column **c 1** and your *area* data in column **c 2** as shown below. Press F2 to get to the Plot Setup menu and then F1 to define the plot. Enter the values as shown in the windows below.









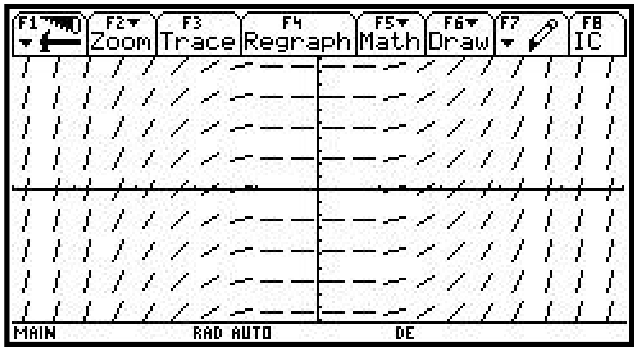
c. On the window [-1, 7] by [-2, 2], sketch your data.

d. What do you notice about the data pattern? Do you recognize a function that fits the data? Try graphing several functions until you have one that appears to fit the data and sketch it over top of your scatter plot from part (c).

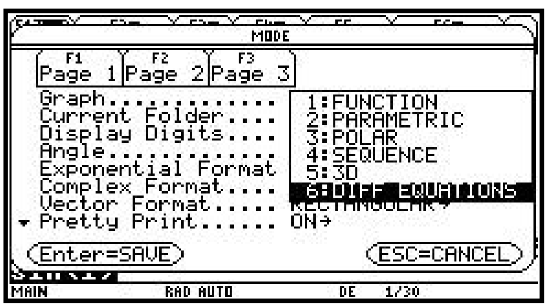
e. What is your function? Compare it with other groups in the class and discuss similarities or differences.

2. Up to now we have spent a great deal of time in calculus thinking about slopes of graphs. Is there any connection between the idea of area under a curve and the idea of slopes of functions? Suppose we think of the function $f(x) = \sin x$ as defining the slope of some function. In other words, if we plug in a value, say $x = 1$, and get that $f(1) \approx .84147$ this could be the slope of some other function. If we did this for many x values and then plotted short segments to represent the numerical slope values in a graphical form we would generate what is called a *slope field*. For example, if we used a function $g(x) = x^2$ as the slopes of some other function, $G(x)$, we would get slopes of 9, 4, 1, 0, 1, 4, and 9 for corresponding x values of –3, -2, -1, 0, 1, 2, and 3 respectively. If we did this for

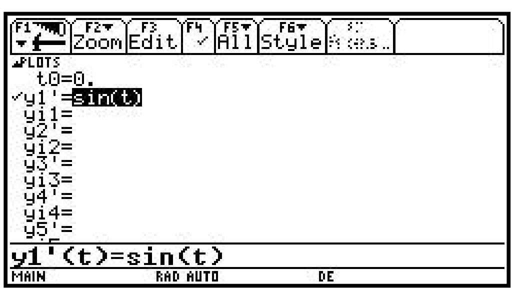
many *x* values and plotted short slanted segments for each slope we would get an image like:

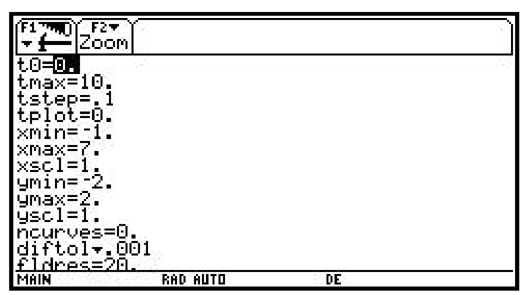


Now consider the function $f(x) = \sin x$ you were working with earlier. Generate a slope field using *f* to define the slopes of some other function, $F(x)$. To do this on the TI-92plus/89 or Voyage 200, you must first set your graphing mode to **DIFF EQUATIONS**. To do this press the [MODE] key and set your modes the following way:



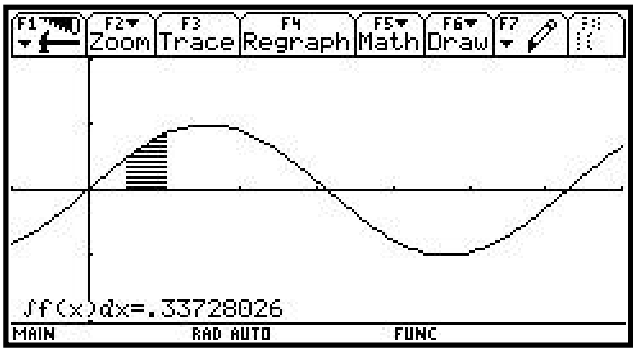
Then press the [Y=] key and enter the slope function as shown below. Finally set your graphing window by pressing the [WINDOW] key and following the screen image below.





a. Now press [GRAPH] and sketch your slope field on the window [-1, 7] by [-2, 2].
b. Take the graph of the function you found in parts 1c and 1d and sketch it over top of the slope field above. What do you notice? Explain any observations you find.

3. Now repeat section 1 of the lab using a *new starting point* of 0.5 for calculating area and record your data below.



x	Area up to x
0.5	0
1	.33728026
1.5	
2	
2.5	
3	
3.5	
4	
4.5	
5	
5.5	
6	
6.5	

a. After you have found a new function that appears to fit your data, sketch it on top of your other function and slope field from part (2). What do you notice? Explain your findings.

You have just used the integral to define a function that we call the *area function*. It can be written as $A(x) = \int_0^x \sin t \, dt$. In general, we can use the integral to define an area function beginning at any point on the x-axis, call it a, and compiling the area under the curve up to x as $A(x) = \int_a^x f(t) \, dt$.

b. How might the area function you just found be affected if we change the value of a in $A(x) = \int_a^x f(t) \, dt$? Explore this question one more time for a new value, a, of your own choice and report your findings.

4. Summarize your findings from this lab with respect to the relationship between area functions and antiderivatives.