Assignment 2
Form and Function

In order to begin a fruitful discussion of mathematical concepts and their relationship to teaching and learning, it is helpful to first examine our own understanding of the concepts in question. In this particular section, we will investigate the concept of inverse function. From a constructivist perspective, no two people have exactly the same conceptual understanding of inverse function. While there may be some common definitions and representations used for describing this idea, not all people have the same mental constructs and images that emerge when the term “inverse function” is mentioned. The way in which other mathematical ideas are connected to this concept may also be quite different depending on the person you ask. Here you will be asked to give some preliminary responses to questions. To begin, in your own words, answer the following questions.

1. In relation to the concept of function, describe what the term “inverse function” means?

2. Describe how you came to understand the concept of inverse function.

3. Imagine yourself in your first teaching position where you have to teach students about inverse functions, how would you teach it? How is it connected to other mathematics they have learned?
Now that you have started to reflect on the concept of inverse function, consider the following classroom exchange as a teacher (Sarah) interacts with her students. Following the vignette, you will be asked to respond to questions surrounding the discussion.

**Case Study: Inverse Functions**

Sarah’s objective for the instructional period was to introduce the topic of inverse function. She began her lesson by asking students if they remembered how to “find the composition of two functions,” as she had discussed the topic only the day before. One student volunteered,

Sue: It is when you write the two functions and then you think of the second one as like the value of the first one— Umm, can I give you an example?

Sarah: You are on the right track Sue, why don’t you tell me what you are thinking and I will put it on the board—

Sue: Okay, let’s say we have \( f(x) = x^2 \) and then \( g(x) = 2x + 1 \), now say we want to find \( f \circ g \) then we say \( f(g(x)) \) equals and then for \( x \) in \( x^2 \) we put \((2x + 1)\)—

Sarah: Very good— Aha!

Sue: So we get \((2x + 1)^2\).

Sarah: What do we do to this now?

Sue: We foil it?

Sarah: Excellent— did you all see what she did— That’s how we find the composition of two functions— Now, if we were asked to find \((g \circ f(x))\) what would we do? (pause) Jack?

Jack: umm

Sarah: Do you need to think about it?

Jack: Uh huh!

Sarah: Okay— (pause)— Randy?

Randy: I think we plug \( x^2 \) for \( x \) in \( g(x) \), right?

Sarah: That’s right— [she turns around and erases the board]— Let’s do one more of these together just to make sure we know what we’re doing (she writes \( f(x) = 3x^2 - 4 \), \( g(x) = \frac{1}{x} \) on the board)— Now I want everyone to find both the \( f \circ g(x) \) and \( g \circ f(x) \)— You have about 5 minutes to do this. If you have questions, raise your hands—
Tanya: I have a question about homework.

Sarah: I will get to that later.

As several students raised their hands for assistance, Sarah decided to save time and to do the problem again on the board so that everyone could see the solution method. She solved both questions for students. At this point Sarah looked at her watch and noticed that she has lost 20 minutes of her instructional time. She announced that she needed to proceed with a new topic.

Sarah: Now, today I want to talk to you about inverse functions... (Erasing the board as she talks) Who knows what I mean by inverse function?

Sue: Is it like one over a function?

Sarah: (laughs) I knew you were going to make that mistake— A lot of people think that when you say inverse function it is like fractions and when we found the reciprocals of them... NO, this is a little different... (pause)... I am going to give you a definition and I want everyone to write this in their notebooks... (she turns around to start writing on the board)— If you get a little confused by the definition don’t worry I will explain it to you after I am finished writing. [She writes the definition directly from the book and reads it aloud as she is writing on the board)—

Sam: I don’t get it!

Sarah: I told you it might be hard for you to understand, let me finish it then I will explain (she finishes writing). Okay, now— It is like this— Say I give you a function like: \(5x + 1\) and then I ask you to find the inverse of this function. The first thing you need to do is to write this as \(y = 5x + 1\), then like we did for linear equations solve it for \(x\), in this case, we first subtract the 1 then we divide by 5, and what do we get? \(\frac{y - 1}{5}\), this is the inverse of our function— To see if we are right, all we have to do now is to compose this function on the original one— If we get \(x\), then it means that we are right and that this one is the inverse function— So, let’s do it.

Peter: I don’t get it— Why do we have to get \(x\)?

Sarah: It’s okay, I will review what I did in a minute and then do a couple of more examples.

[Peter is completely disengaged and starts throwing his pencil at the person sitting next to him. Sarah notices this and asks Peter to pay attention.]
Reaction to Vignette

1. Describe the central issues involved in the classroom exchange.

2. At one point a student offers up an explanation of what is meant by an inverse function. In response to this, Sarah (the teacher) says, “I knew you were going to make that mistake— A lot of people think that when you say inverse function it is like fractions and when we found the reciprocals of them... NO, this is a little different...”. Discuss your thoughts on Sarah’s response.

At this point we will take some time to explore some mathematical ideas using an activity called, Form and Function. This activity can be found in Module 1 of these materials. To begin, form groups of 3 or 4 people and follow the activity instructions. The materials needed for the activity are a triangle attribute block, transparency pen, and a computer algebra system.
Reaction to Vignette Revisited

Now that you have explored the concepts in *Form and Function*, revisit the classroom vignette on inverse functions. This time, try to view the vignette through the lens of undergraduate mathematics. In particular, try to relate your experiences from *Form and Function* to what is happening in the classroom.

1. In light of your experience in our activity, *Form and Function*, describe the central issues involved in the classroom exchange.

2. Recall that at one point in the vignette, a student offers up an explanation of what is meant by an inverse function. In response to this, Sarah (the teacher) says, “I knew you were going to make that mistake—A lot of people think that when you say inverse function it is like fractions and when we found the reciprocals of them... NO, this is a little different...”. In light of your experience with *Form & Function*, discuss your thoughts on Sarah’s response.
3. In light of your experience in our activity, *Form and Function*, how would you handle the same situation in your classroom? Give specific examples to illustrate what you would do.

4. How would you have dealt with Peter’s comment, “I don’t get it— Why do we have to get $x$?”?