A Mathematical History Tour

One can invent mathematics without knowing much, if any, of its history. But one cannot have a mature appreciation of mathematics without a substantial knowledge of its history.

—Abe Shenitzer (Kleiner 1988)

Mathematics has a place in history, and history has a place in mathematics. Unfortunately, the rich history of mathematics is seldom included in either history or mathematics courses. It is up to us, as mathematics educators, to learn about the history of our subject both because it is interesting and because it can be useful in our teaching. Three major reasons for bringing the history of mathematics into our classrooms are (1) to put mathematics in its historical context; (2) to show the interaction of mathematics with the cultures of which it has been a part; and (3) to shed light on the learning—and therefore teaching—of mathematics.

THE CONTEXT OF MATHEMATICS

Mathematics has been done by real people, some of whom were very interesting in addition to being successful in other fields. Integrating biographies of mathematicians into the relevant content is an effective way to introduce students to these people. Students remember material better when they can relate it to a name, place, or anecdote. For example, when studying the Cartesian coordinate system, students may be interested to learn that Descartes was a well-dressed, eccentric figure whose wealth allowed him to stay in bed until 11:00 A.M. every day. He came to the attention of Queen Christina of Sweden, who invited him to tutor her. Descartes refused her request for a year, until the queen sent a battleship for him. Christina required Descartes to instruct her every morning from five o’clock to six o’clock, during the Scandinavian winter, no less. After only eleven weeks of this regimen, Descartes died of influenza (Bergamini 1963).

Biographies of female mathematicians can show that women have been involved in mathematics from its earliest recorded history to the present. Hypatia was a fourth- and fifth-century Greek mathematician who contributed to the algebra of her day. Despite several offers, she never married, claiming that she was “wedded to the truth” (Osen 1974, 29). One day, on her way to teach classes, Hypatia was dragged from her chariot by a religious mob and tortured to death. Today, an increasing number of resources discuss female mathematicians past and present. The bibliography lists several sources.

Mathematics of other times and places can be used to address multicultural issues. Mathematics is evident in the arts and crafts of many cultures. Illustrations of such geometric properties as symmetry, congruence, and perspective are found in the weaving, beadwork, pottery, and basketry of Native American and African peoples. Islamic mosaics and other artworks are often geometric and exhibit these same properties. Students’ mathematical experience can be enriched by examining or designing quilts or beadwork and describing the symmetry or other geometric properties of the artifact.

Both ancient and primitive art display an interesting lack of perspective. A fifteenth-century Italian painter named Brunelleschi used mathematical principles to conduct scientific experiments with perspective. His work, which was continued by other Renaissance artists, revolutionized painting, sculpture, and architecture. Contrasting pre-Renaissance and Renaissance art would indicate to students the importance of perspective in accurately representing our world.

Students might be interested to know that our system of numeration is not the only one in use. Many tribes in western Africa use a base-twenty system. And, of course, our modern electronic computers use the binary system. Many early systems, such as those used by the Egyptians and Mayans, did not use bases at all but instead had detailed symbols. Expressing large numbers was quite cumbersome. Many African tribes used elaborate finger gestures for counting. Later societies used piles of...
shells or stones to carry out everyday calculations. Our modern term *calculator* is from the Latin *calculus*, which means pebble (Coughlin and Zitarelli 1984).

Our system, the Hindu-Arabic, was developed in India, and Arabs spread it to North Africa and Europe. It was not well accepted at first. In fact, in 1299 C.E. a law was passed in Florence requiring the bankers of that city to use Roman numerals rather than Arabic in their business to prevent forgeries. Merchants and traders of the Mediterranean area adopted our decimal system for its ease of calculation and eventually popularized it. Having students perform computations in Roman numerals might convince them that our number system is very efficient. Asking them to express decimals in Roman numerals—or in other systems without place value—would make clear to them the importance of zero.

Another interesting area of the history of mathematics is the origin of some of the symbols that we use. The “=” sign was introduced by the English mathematician Robert Recorde in 1500 “bicause noe.2. [sic] thynges, can be moare equalle” (NCTM 1989, 244). Thomas Harriott, a surveyor who accompanied Sir Walter Raleigh on an expedition to North America in 1585, wrote a book titled *A Brief and True Report of the New-found Land of Virginia* (now North Carolina), which contained his observations of this new land. One drawing of a Native American male showed a design on his back. Harriott considered this design beautiful and incorporated two of its symbols, “>” and “<,” into his mathematical writings. See figure 1. Besides aesthetics, Harriott must have been struck by the resemblance that those symbols have to the notions of greater than and less than. These symbols were rejected at first but gradually became accepted (Coughlin and Zitarelli 1984). When introducing these symbols, ask students to guess which one means greater than and which means less than.

**INTERACTION WITH CULTURES**

The mathematics that is done by the people of a culture both reflects and affects that culture. Comparing the mathematical traditions of two very different cultures, the ancient Greek and Chinese, illustrates these points.

The Chinese used mathematics primarily to solve practical problems. The government—which was powerful, centralized, and bureaucratic—employed most of the country’s mathematicians. The problems on which these mathematicians worked were concerned with such activities as building walls, moving and feeding the vast army, farming, and equitably dividing estates. Mathematicians developed the imperial calendar and made astrological predictions. Mathematics existed to further the technology of the Chinese culture, and that technology was impressive. Hydraulic engineering, bridge building, navigation, military techniques, and medical procedures were all very advanced.

In some areas, Chinese mathematics was the most advanced in the world. The Chinese used irrational and negative numbers earlier than Europeans. They developed methods for solving equations up to the third degree, including matrix methods for solving systems of linear equations. They used what we now call *Pascal’s triangle* to compute binomial coefficients (Downing 1990).

In addition to the Chinese question of how, the ancient Greeks asked the modern scientific question of why. Greece had a wealthy ruling class that owned slaves and therefore enjoyed a great deal of leisure. This class produced intellectuals, scholars, and scientists, that is, people who could devote their lives to pursuing knowledge. Greek mathematicians were more academic than bureaucratic. The intellectual atmosphere that these people created was conducive to the development of the foundations of mathematics. Their attitude toward the world was rational and critical, knowledge was more for understanding than for utility. They believed that nature was orderly and that they could understand it. Greek mathematicians had no imperative to apply their mathematical discoveries.

Greek mathematics is exemplified by Euclid’s work. In his *Elements*, Euclid began with a set of
definitions and axioms and proved all other geometric facts using deductive logic alone. For example, he proved the visually obvious fact that the sum of the lengths of any two sides of a triangle is greater than the length of the third side. His proof was ridiculed by some of his contemporaries, who claimed that even a dumb animal knew this theorem, for "if fodder is placed at one point and the ass at another, he does not, in order to get his food, traverse the two sides of the triangle but only the side separating them" (Coughlin and Zitarelli 1984, 306). The ass, however, could not prove this truth, but Euclid could and did. This insistence on proof, the axiomatic method, is the Greek legacy to modern mathematics.

The mathematics of many other ancient cultures resembled that of the Chinese more than that of the Greeks. Before 2000 B.C.E., communities along the rivers of Africa, Asia, and North America used mathematics as a practical science to facilitate computation of the calendar, tax collection, and government administration. Most mathematical activity of the ancient Egyptians seems to have been concerned with agricultural issues, such as making bread, feeding animals, and storing grain. We know little of the mathematics of ancient Eastern cultures because, except for Egyptian papyrus, much of the mathematics of these peoples was done on less durable materials and has either been lost or is difficult to date.

We do know that the ancient Greek and Chinese viewpoints exemplified both the theoretical and applied aspects of mathematics. An aside to these two approaches is the way that the traditional curriculum has treated algebra and geometry. Geometry was developed axiomatically by ancient Greek mathematicians. For years we have taught geometry in this way, emphasizing proofs over computations. In contrast, until the middle of the nineteenth century, algebra lacked an axiomatic foundation. Even today, below the college level, students learn algebra by doing it, with some justifications for what is done but certainly not with the level of rigor that we still find in geometry classrooms.

HISTORY AND LEARNING
Besides classroom use, history has more to teach us. Parallels are observed between the historical development of mathematical concepts and the way that they develop in an individual mind. History suggests ways of teaching. As George Pólya said, “Having understood how the human race has acquired the knowledge of certain facts or concepts, we are in a better position to judge how the human child should acquire such knowledge” (Pólya 1962, 132).

In particular, studying the history of a given concept within the mathematical community may offer clues as to why that concept is not easy to understand. A connection exists between subject matter that is difficult for a student to grasp and that which was difficult for the mathematical community to accept.

The development of number systems is a good example. In historical order, the numbers that developed were the natural numbers, rational numbers, zero, negative numbers, irrational numbers, and complex numbers, not to mention such numberlike creatures as matrices. This sequence is similar to the order in which we teach these number systems in our classrooms.

Thinking about how we use numbers in our lives is interesting. Counting was our first mathematical activity, and counting uses the natural numbers. We also make some use of rational numbers. Some kinds of numbers, the ones that developed later, are used only in mathematics classes. Zero and negative numbers have been troublesome concepts for mathematicians and did not achieve full acceptance in the mathematical community in the West until the last five hundred years; even when they were first accepted as numbers, they were not accepted as roots of equations. Pascal said, “I have known those who could not understand that to take four from zero there remains zero” (Kline 1980, 115). As recently as 1831, De Morgan stated that the expression 0 – a is “inconceivable” (Kline 1980, 155).

The Pythagoreans realized that solving right triangles could lead to what we now call irrational numbers. They suppressed this discovery because they believed that all numbers could be expressed as the ratio of two integers. Legend says that colleagues drowned the man who first described irrational numbers.

We often play mathematics games in my classes, and I tell my students, “I’m thinking of a number between one and twenty . . . ” to determine who goes first. Students invariably guess integers, and I just as invariably choose such a number as \( \sqrt{7} \) or \( \pi \).

Students laugh, but I want to make the point that my numbers are every bit as valid as theirs.

In 1545 Girolamo Cardano, an Italian mathematician who was also a gambler and philosopher, published a book in which he described an algebraic method of solving cubic and quartic equations. In his book, Cardano also attempted to solve the problem of finding two numbers whose sum is 10 and whose product is 40. This problem led to the quadratic equation \( x^2 – 10x + 40 = 0 \). The solutions to this equation are \( 5 \pm \sqrt{-15} \). Cardano discarded the solution, calling it “as subtle as it is useless” (NCTM 1969, 245). However, when Cardano applied his method of solving cubics to the equation \( x^3 = 15x + 4 \), he found that it has three real solutions, yet the algorithm for finding them uses the square root of a
negative number (Kleiner 1988, 584). This finding was not so easy to dismiss.

Thirty years later, another Italian mathematician, Rafael Bombelli, continued Cardano’s work and defined arithmetic operations on complex numbers. However, mathematicians did not consider these numbers legitimate for nearly three hundred years after Bombelli. We usually introduce complex numbers in the algebra curriculum in order to solve quadratic equations, yet historically, their first practical use was in solving cubic equations.

Many of the concepts that we take for granted have had a long and winding road to respectability. Yet we ask students to accept in a short period of time concepts that sometimes took centuries to be accepted by the mathematical community, even though students lack the mathematical foundation and expertise possessed by those mathematicians. Perhaps noting the difficulty that great mathematical thinkers had with such concepts can make us more understanding of students who give in to the temptation to divide by zero or are confused by operations on signed numbers, as well as to marvel at our own competence in those areas.

I hope that you have enjoyed your mathematical history tour. Like any tour, it includes only a few highlights, for the history of mathematics is a vast subject. I hope that I have whetted your appetite for further study of this fascinating subject. The following resources may help you begin or continue your study.

**BIBLIOGRAPHY**

Readers interested in more information about the history of mathematics and mathematicians will find it in the following:


